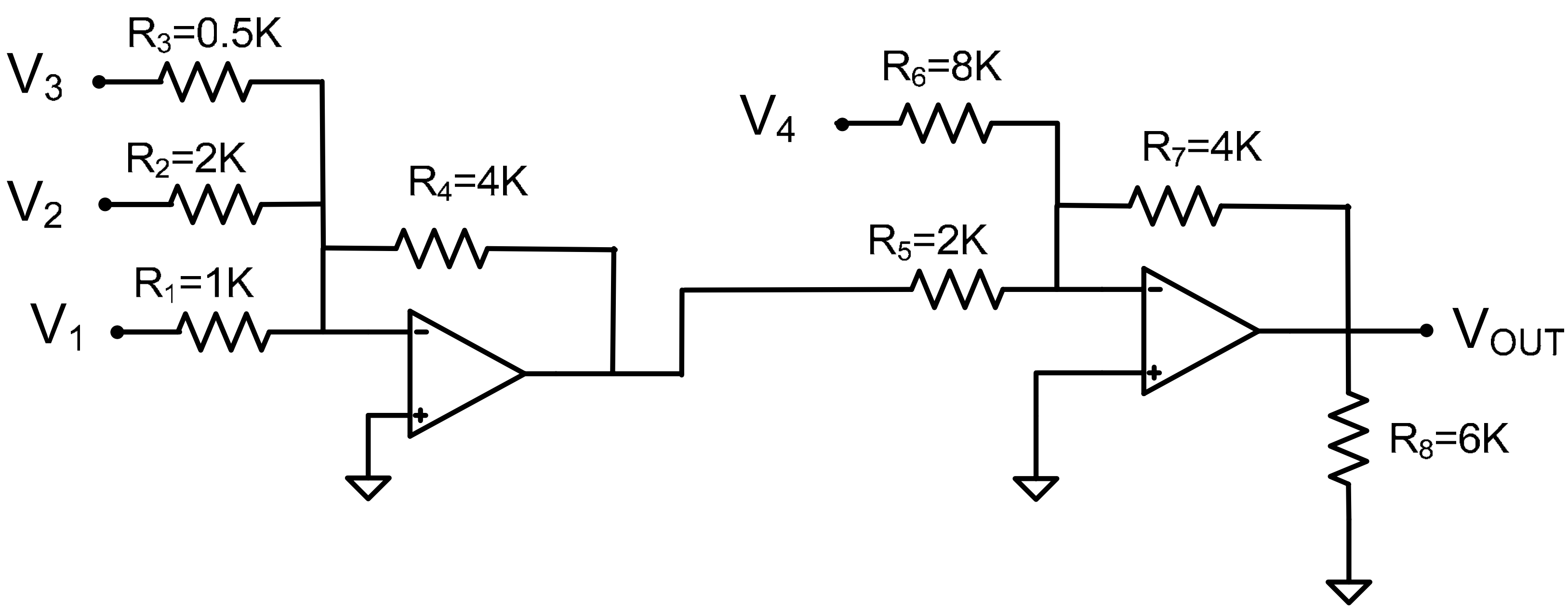
EE 230
Lecture 12

Basic Applications of Operational Amplifiers

- Generalized Impedances
- Integrators
- Analog Computation

Quiz 9

Determine the output voltage for the following circuit. Assume the op amps are ideal.



And the number is ?

1 3 8

5 4

2 6

9 7

And the number is ?

1

3

8

5

4

2

5

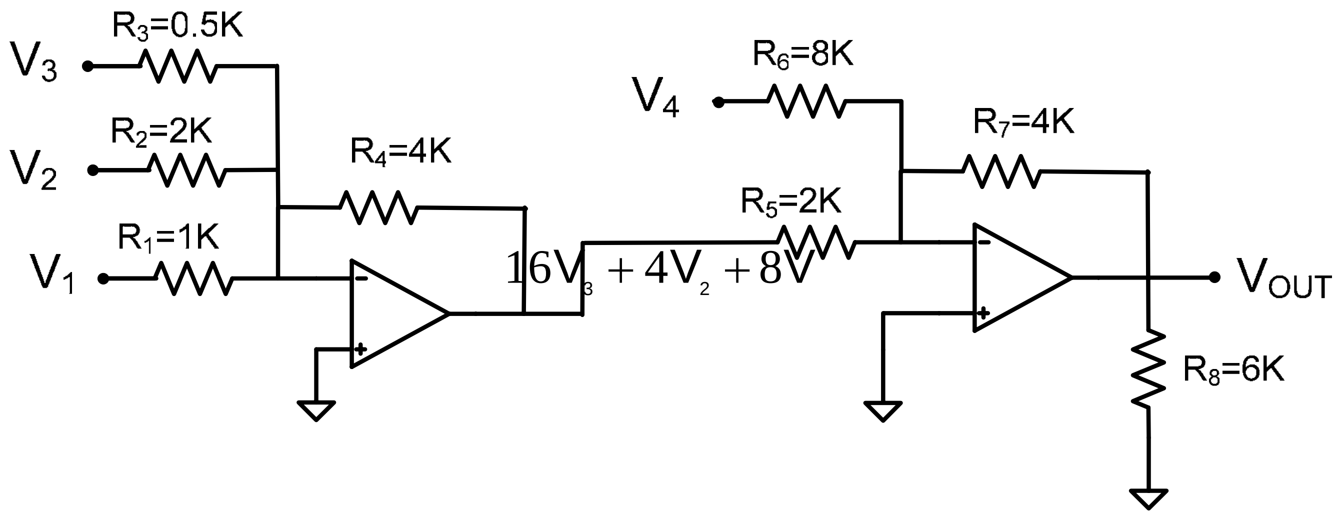
6

9

7

Quiz 9 Solution:

Determine the output voltage for the following circuit. Assume the op amps are ideal.



By superposition

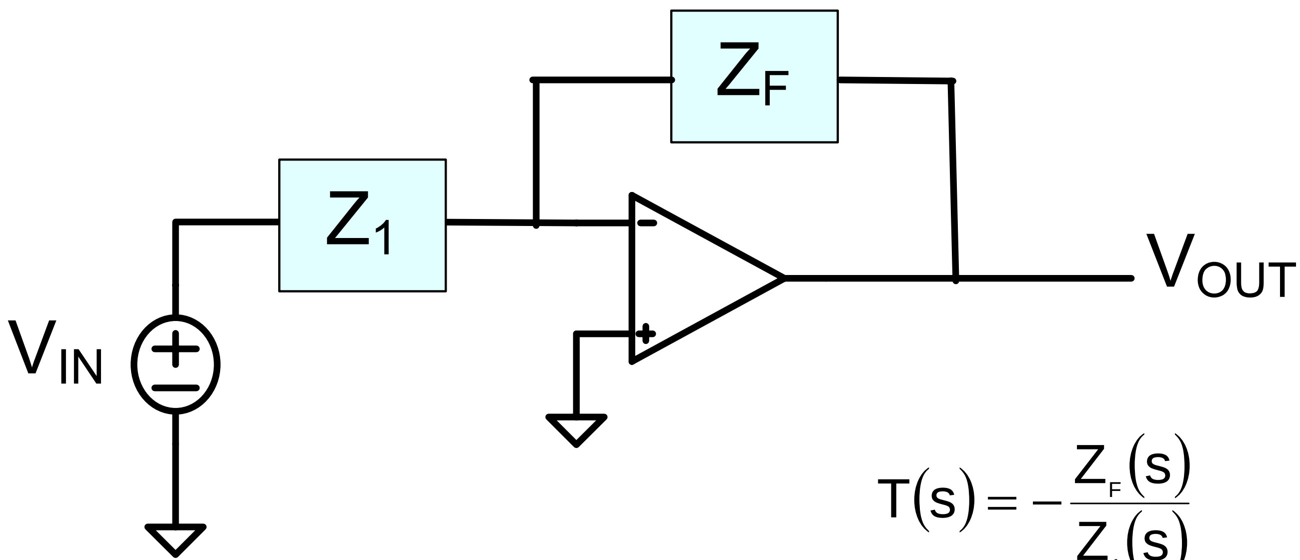
$$V_{OUT} = -\frac{R_7}{R_6}V_4 - \frac{R_7}{R_5}\left(-\frac{R_4}{R_3}V_3 - \frac{R_4}{R_2}V_2 - \frac{R_4}{R_1}V_1\right)$$

$$V_{OUT} = -0.5V_4 + 16V_3 + 4V_2 + 8V_1$$

Review from Last Time

Summing Amplifiers with mixed weights on inverting and noninverting inputs can be readily obtained

Integration and Differentiation functions can be obtained from the basic generalized feedback amplifier structure



$$T(s) = -\frac{Z_F(s)}{Z_1(s)}$$

Integration and Differentiation

Integration

$$y(t) = K \int_{\tau=0}^{t} x(\tau)\, d\tau$$

$$\mathcal{L}(y(t)) = \frac{K}{s} \mathcal{L}(x(t))$$

$$Y(s) = \frac{K}{s} X(s)$$

Integration and Differentiation

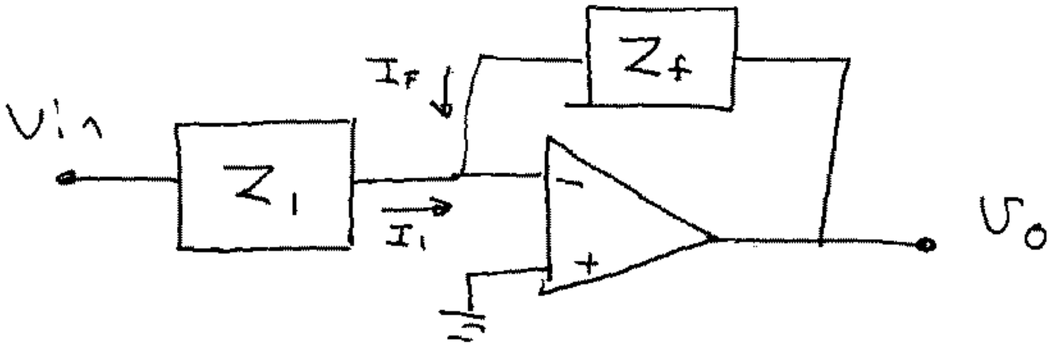
Differentiation

$$\mathrm{y}(\mathrm{t}) = \mathrm{K}\frac{dx(t)}{dt}$$

$$\mathcal{L}(\mathrm{y}(\mathrm{t})) = \mathrm{Ks}\mathcal{L}(\mathrm{x}(\mathrm{t})) - x(0)$$

$$\mathrm{Y}(\mathrm{s}) = \mathrm{KsX}(\mathrm{s}) - x(0)$$

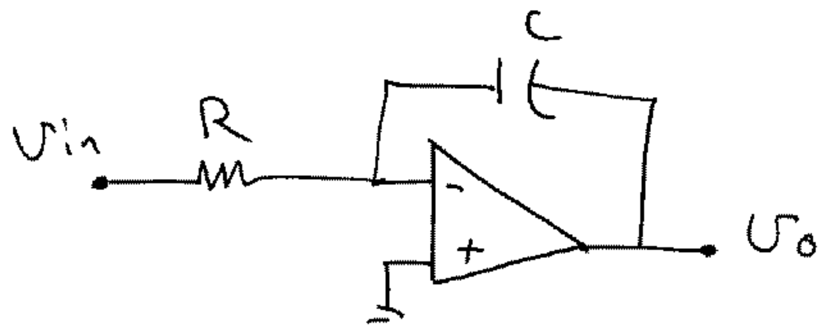
Generalized Inverting Amplifier



S-domain impedances

$$\left.\begin{aligned} I_1 &= \frac{V_{in}}{Z_1} \\ I_F &= \frac{V_o}{Z_f} \\ I_1 &= -I_F \end{aligned}\right\} \quad \frac{V_o}{V_i} = -\frac{Z_f}{Z_1}$$

what if $Z_f = 1/sC$, $Z_1 = R$



$$\frac{v_o}{v_i} = -\frac{1}{RCs}$$

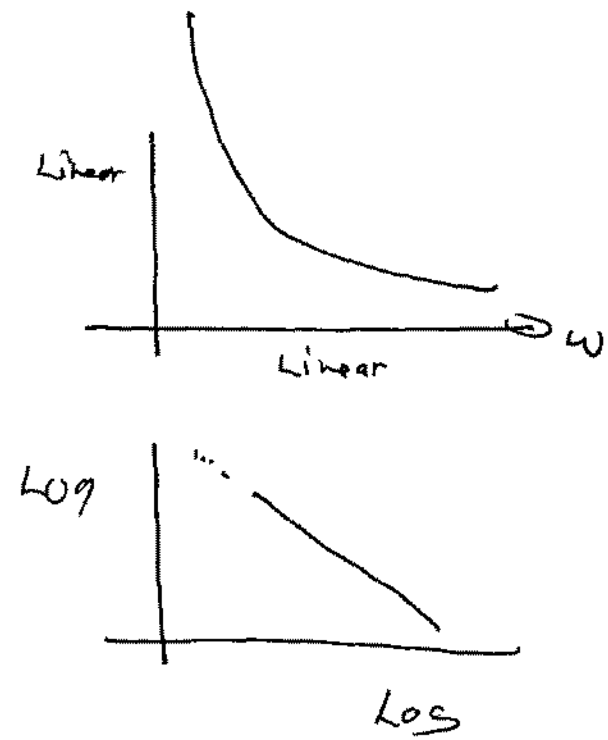
what is this?

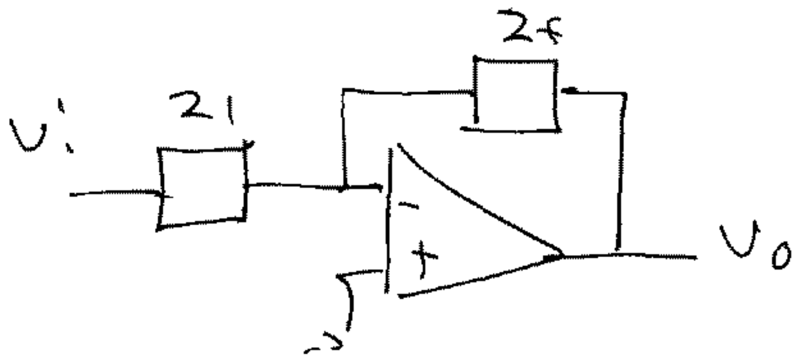
$$T(s) = -\frac{1}{RCs}$$

$$T(j\omega) = -\frac{1}{j\omega RC}$$

$$|T(j\omega)| = \frac{1}{\omega RC}$$

Integrator Gain





$$\frac{v_o}{v_i} = -\frac{Z_f}{Z_1}$$

Let $Z_1 = \frac{1}{sC}$

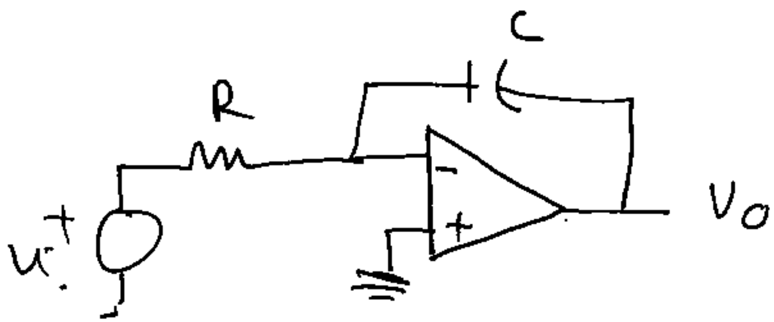
$Z_f = R$

$$\therefore \frac{v_o}{v_i} = -\frac{sCR}{\cancel{R}}$$

Differentiator (Inverting)

- Not widely used
- Noise relentlessly amplified
- stability problems with implementation

Inverting Integrator



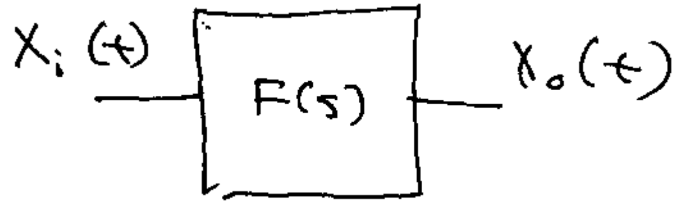
$$\frac{V_o}{V_i} = -\frac{1}{RCS}$$

Inverting because gain is negative

$$R_o = 0$$

$$R_{in} = R$$

- Seldom used in open-loop application
- widely used in feedback application



If $X_i(t)$ has any dc component

$$F(s) = 1/s$$

$$X_o(t) = \int_0^{\infty} X_i(t) = \int_0^{\infty} \left(A_0 + A_1 \sin(\omega t + \theta_1) + \cdots \right)$$

$$= \int_0^{\infty} A_0 \, dt + A_1 \int_0^{\infty} \sin(\omega t + \theta_1) + A_2 \int_0^{\infty} \sin(2\omega t + \theta_2) + \cdots$$

$$\lim_{t \to \infty} A_0(t)$$

diverge to ∞

- Integrator function is ill-conditoned for open loop applications

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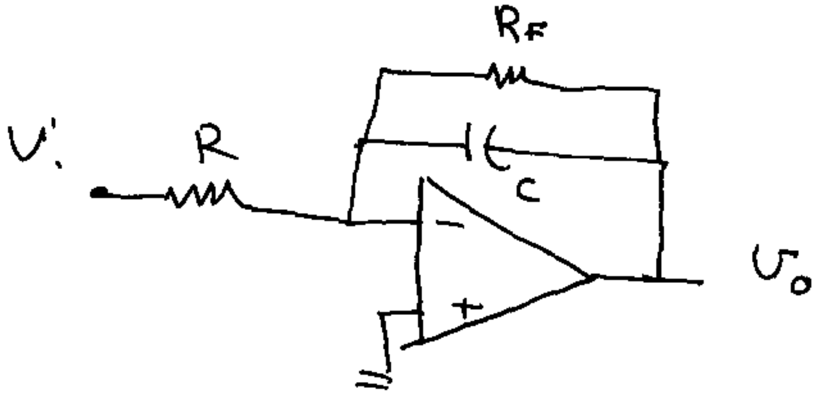
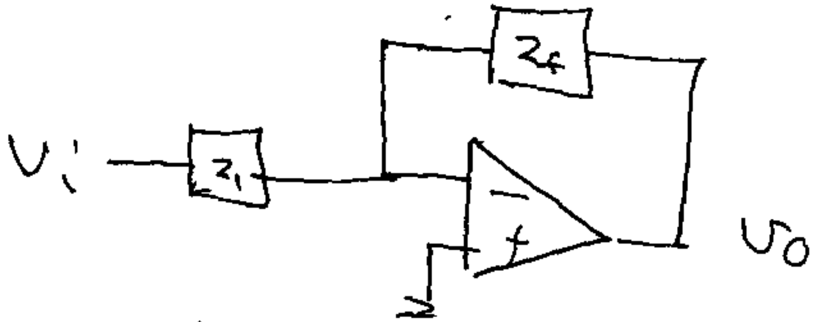
Method for approx. open loop integration



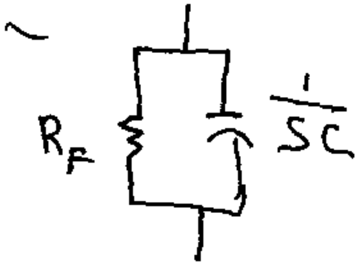
- R_F very large

$$V_o \simeq -\frac{1}{RC}\int_0^t V_i(t)\,dt + V_i(0)$$



Lossy Integrator

$Z_f \sim$



$Z_1 \sim$

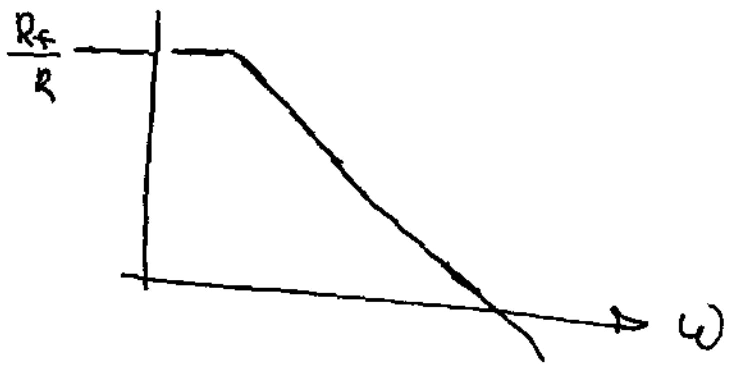
$$Z_f = \frac{(R_f)^1/_{SC}}{R_f + {}^1/_{SC}} = \frac{R_f}{1 + R_f C S}$$

$$\frac{V_o}{V_i} = -\frac{Z_f}{Z_1} = -\frac{R_f}{(1 + R_f C S)R}$$

Ideally $R_f = \infty$

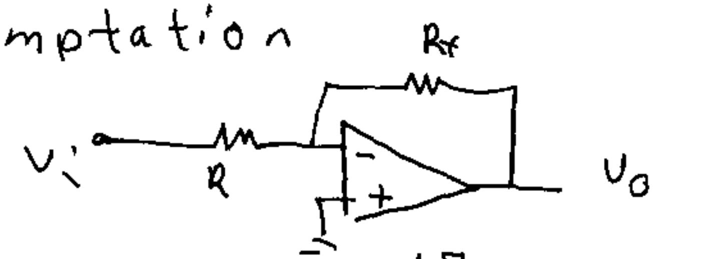
$$\frac{V_o}{V_i} = -\frac{1}{RCS}$$

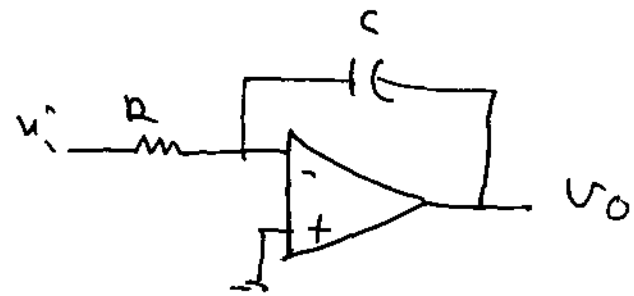
$$\left|\frac{V_o}{V_i}\right| = \frac{R_f/R}{\sqrt{1 + (R_f C \omega)^2}}$$



Noninverting Integrator.

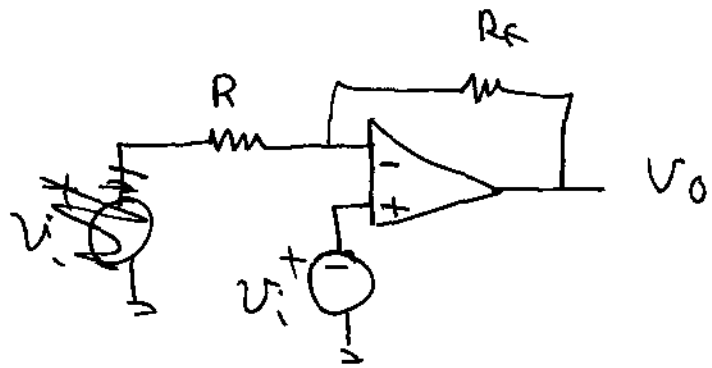
Temptation

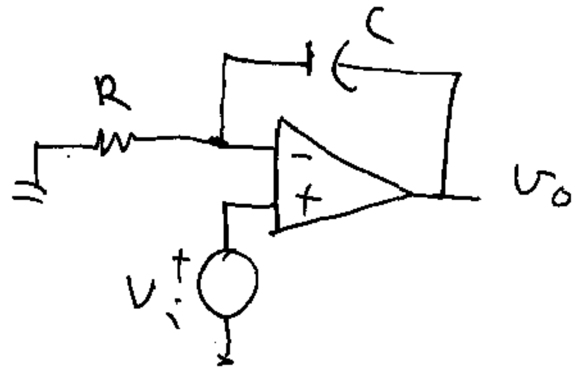




$$\frac{v_o}{v_i} = -\frac{R_f}{R}$$

$$\frac{v_o}{v_i} = -\frac{1}{RCs}$$





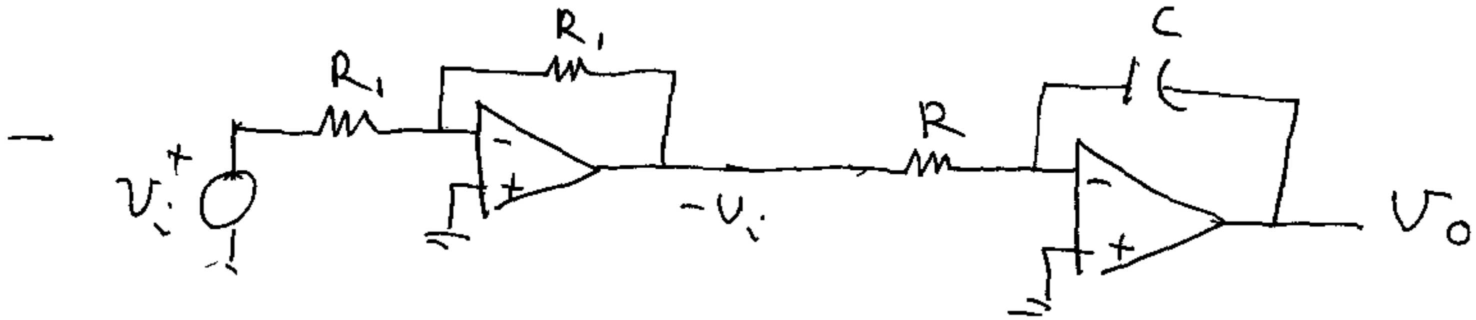
$$v_i = v_o\left(\frac{R}{R + 1/sC}\right)$$

$$v_i = v_o \frac{RCs}{1 + RCs}$$

$$\therefore \frac{v_o}{v_i} = \frac{1 + RCs}{RCs}$$

This is not ~~a~~ a noninverting integrator

Noninverting Integrator

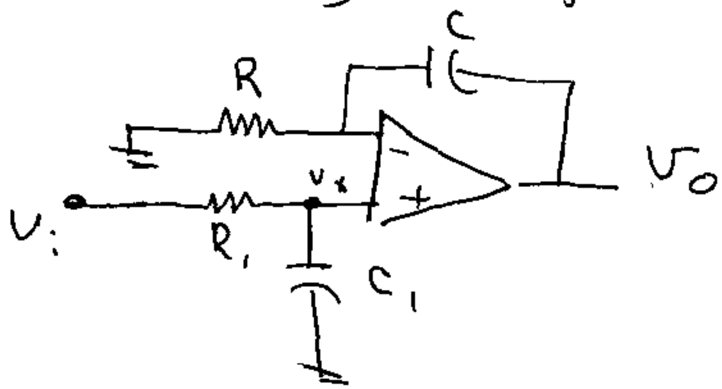


$$v_o = \frac{1}{RCs} v_i$$

- follow inverting integrator with inverter instead

- Most popular noninverting integrator structure

Noninverting Integrator



$$V_x = \frac{\frac{1}{sC_1}}{R_1 + 1/sC_1} V_i = \frac{1}{1+R_1C_1S} V_i$$

$$V_x = \frac{R}{R + 1/sC} V_o = \frac{RCS}{1+RCS} V_o$$

$$\frac{V_o}{V_i} = \left(\frac{1}{1+R_1C_1S}\right)\left(\frac{1+RCS}{RCS}\right)$$

If $R_1C_1 = RC$ (pole/zero cancellation)

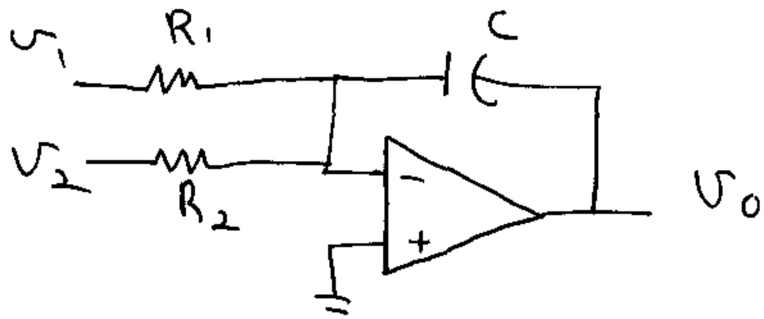
$$\frac{V_o}{V_i} = \frac{1}{RCS}$$

Advantage: eliminated one opamp

Disadvantage: - two capacitors

- precise relationships between $R_1C_1 = RC$

Summing Inverting Integrator.



$$v_0 = -\frac{1}{R_1 C s} V_1 - \frac{1}{R_2 C s} V_2$$

Applications to solving differential equations.

Example:

$$V_o = K_1 \int v_o + K_2 \iint v_o + K_3 V_i$$

standard integral form

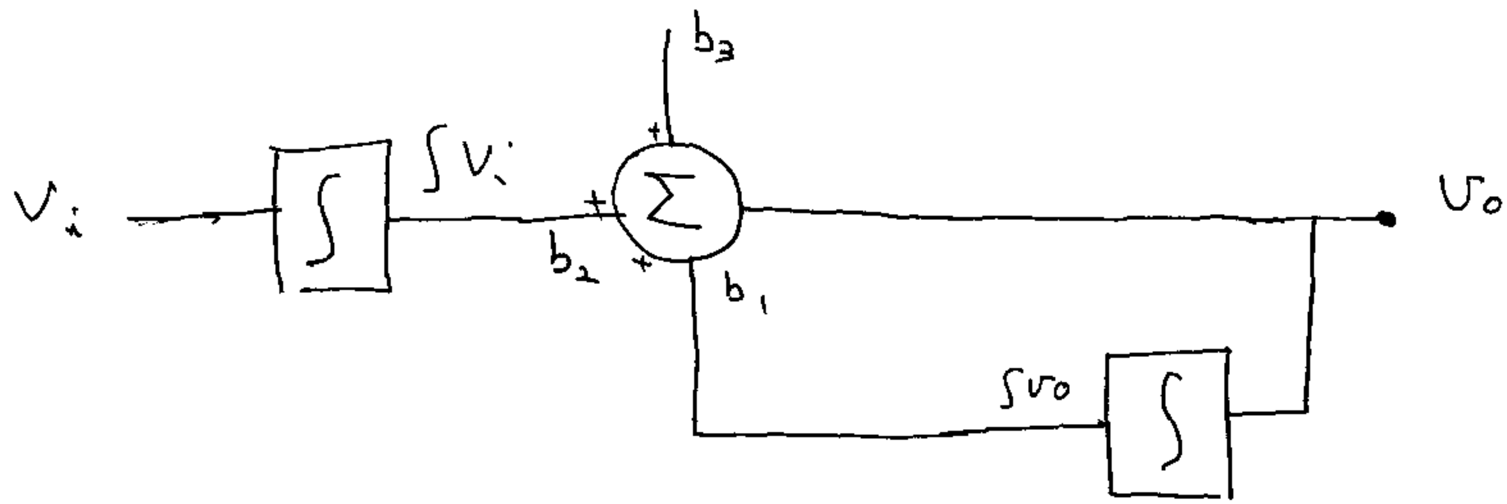
$$v_o' = K_1 V_o + K_2 \int v_o + K_3 V_i'$$

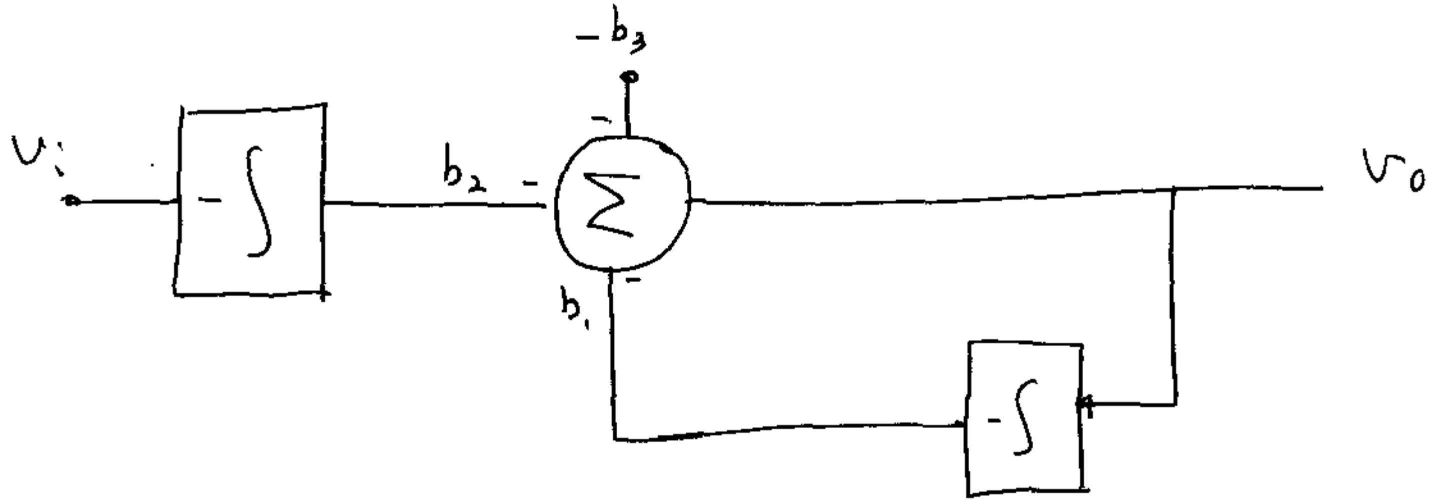
$$v_o'' = K_1 v_o' + K_2 V_o + K_3 V_i''$$

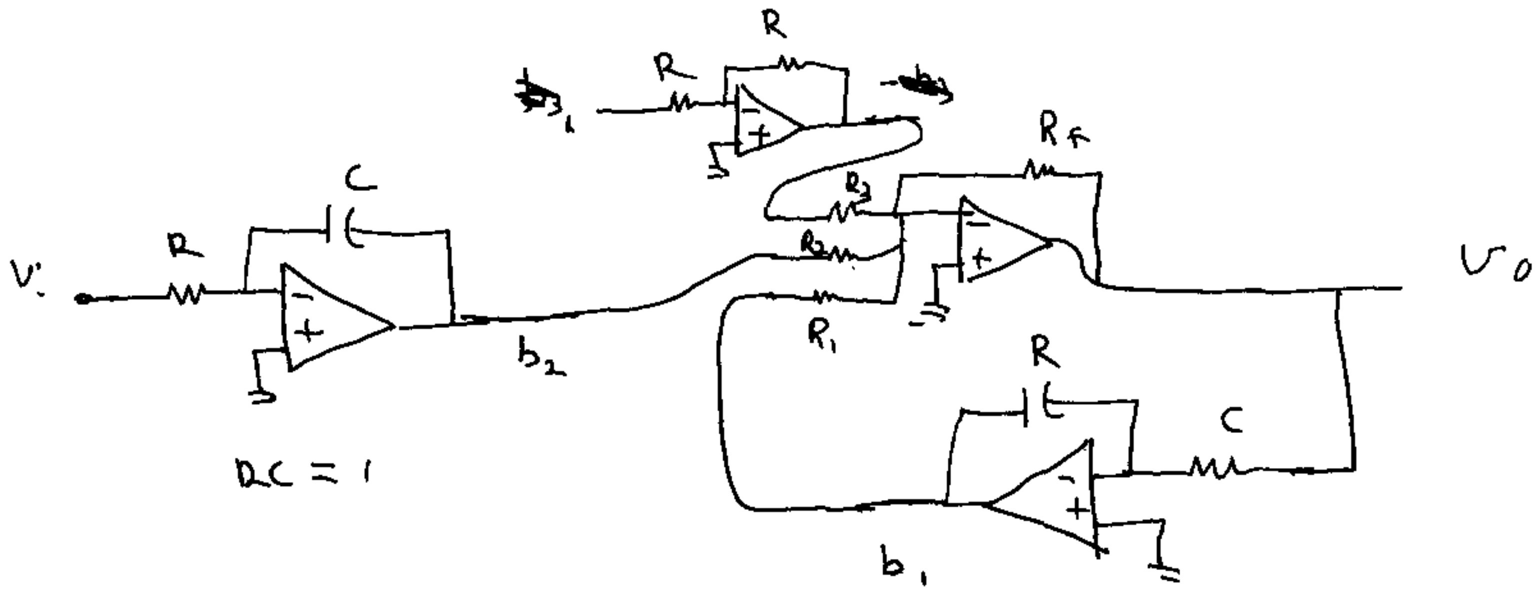
$$V_o = a_1 v_o' + a_2 v_o'' + a_3 V_i''$$

standard differential form

$$v_o = b_1 \int v_o + b_2 \int v_i + b_3$$







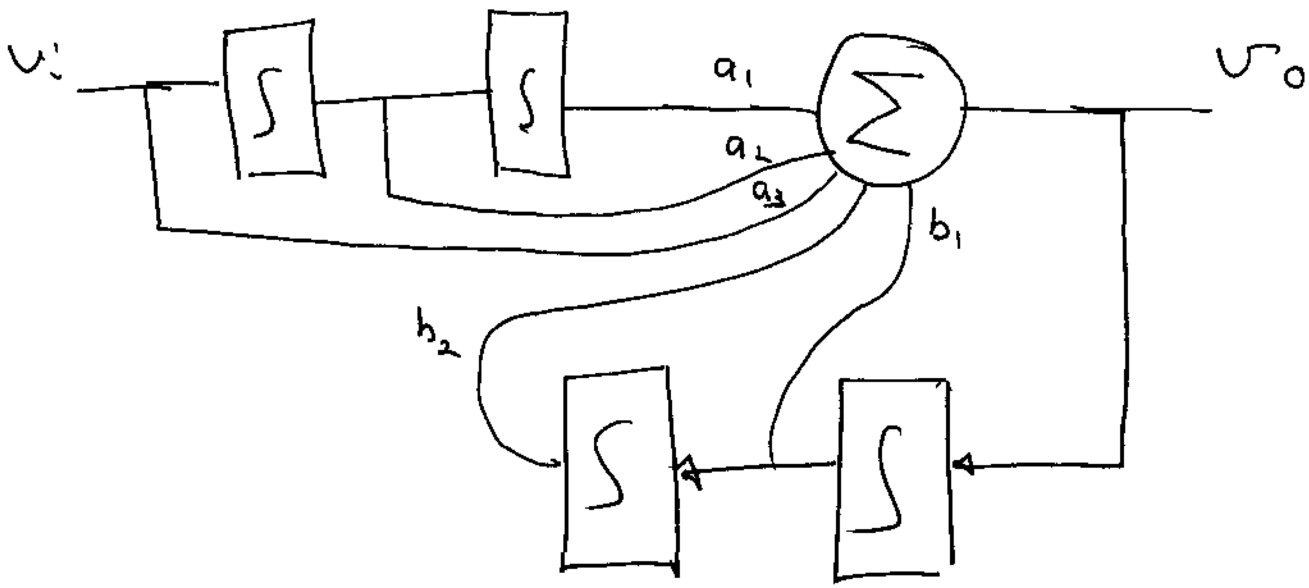
$RC = 1$ $\qquad$ $RC = 1$

$$\frac{R_f}{R_1} = b_1 \qquad \frac{R_f}{R_2} = b_2 \qquad \frac{R_f}{R_3} = b_3$$

- Straightforward to solve an arbitrary differential equation with inverting integrators, summing amplifiers and inverters

- Analog Computer solves differential equations

End of lecture



$$v_o = \frac{a_1 v_i}{s} + \frac{a_2 v_i}{s^2} + a_3 v_i + \frac{b_1 v_o}{s} + \frac{b_2 v_o}{s^2}$$

$$\frac{v_o}{v_i} = \frac{a_3 s^2 + a_2 s + a_1}{s^2 - b_1 s - b_2}$$

any coeff can be positive or negative or zero

Arbitrary transfer function synthesis is easy to achieve.

$$v_o = \frac{1}{b_2} v_o'' - \frac{b_1}{b_2} v_o' - \frac{a_1}{b_2} v_i - \frac{a_2}{b_2} v_i' - \frac{a_3}{b_2} v_i''$$

$$v_o = \alpha_2 v_o'' + \alpha_1 v_o' + \beta_0 v_i + \beta_1 v_i' + \beta_2 v_i''$$

standard differential form